


# Critical Test Appraisal

## Uncertainty of Measurement

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# Teaching goals

Understand requirements of standards

Make sure that the exercise **adds value**

understand certainty

understand the diagnostic process

understand analytical errors

Find **relevant** information **efficiently** and use it **effectively**

Important steps in the life cycle of a method

- validation plan and method validation: analyse

- **measurement principle**

- **robustness of assay design**

- continuous learning:

- **analysis of internal quality control data**

- use knowledge for **method optimization & risk containment**



# Uncertainty of Measurement

## Requirements of the Standards

### ISO 17025 § 5.4.6.

Testing laboratories shall have and shall apply procedures for estimating uncertainty of measurement.

In certain cases the nature of the test method may preclude rigorous, metrologically and statistically valid, calculation of uncertainty of measurement.

### ISO 15189 § 5.6.2.

The laboratory shall determine the uncertainty of results, where relevant and possible. Uncertainty components, which are of importance, shall be taken into account.

Sources that contribute to uncertainty may include sampling, sample preparation, sample portion selection, calibrators, reference materials, input quantities, equipment used, environmental conditions, condition of the sample and changes of operator.

The lab shall have a procedure

The effort shall be commensurate

Preanalytical

Meterological  
Traceability

# Uncertainty of Measurement

## Requirements of the Standards

### ISO 15189 § 5.6.3.

A programme for **calibration** of measuring systems and **verification of trueness** shall be designed and performed so as to ensure that results are traceable to SI units or by reference to a natural constant or other stated reference.

Where none of these are possible or relevant, other means for **providing confidence** in the results shall be applied, including but not limited to the following:

- a) participation in a suitable programme of interlaboratory comparisons
- b) use of suitable reference materials, certified to indicate the characterisation of the material
- c) examination or calibration by another procedure
- d) ratio or reciprocity-type measurements
- e) mutual consent standards or methods which are clearly established, specified, characterized and mutually agreed upon by all parties concerned
- f) documentation of statements regarding reagents, procedures or the examination system when traceability is provided by the supplier or manufacturer

# Philosophy

# 1. Understanding Uncertainty

## certainty

results from (structured) reasoning,  
taking into account all known relevant elements (= state of knowledge).  
Certainty is then the degree of belief (= probability)  
that the facts that are found (= result of a measurement)  
are relevant, so that an action is more likely to follow (= consequential).

## Certainty $\approx$ Relevant (validated) Evidence

(U.S. Federal Rule of Evidence 401)

Our belief can be expressed as a probability:

$L$  (conclusion | the facts that are found)

Our beliefs vary from weak to strong,  
and rarely are absolute (0, 1).

# 1. Understanding Uncertainty

**certainty** results from (structured) reasoning, taking into account all known relevant elements (= state of knowledge). Certainty is then the degree of belief (=probability) that the facts that are found (= result of a measurement) are relevant, so that an **action is more likely to follow (= consequential)**.

The **certainty** that counts follows from (structured) analysis of the state of knowledge, in the head of the clinician.

**The fallacy of the diagnostic process:**

$P(\text{fact} | \text{condition}) \leftrightarrow L(\text{condition} | \text{fact})$

These are used interchangeably in common language, but they are not.

Test characteristics

Post-Test Probability

Bayes-theorema



also dependent on  $P(\text{fact} | \text{condition absent})$ , spectrum & prevalence

Failing to provide *info about test capability* is a source of uncertainty.

Failing to analyse all facts in a structured way is a source of (post-analytical) error.

## 2. Understanding Measurement

**measurand:** particular quantity subject to measurement

1. physicochemical phenomenon that is measured for the **analyte** that is studied
    - report line : dimensions of measured phenomenon ... for analyte ...
  2. state of the phenomenon, body, or substance under study
    - report line : measured in matrix ..., at temperature ..., sampled ... ..
    - reference values : under specified conditions (age, time of day, time post load ...)
  3. The **measurement** can change the quantity that is actually measured
    - e.g. potential difference between the terminals of a voltometric element depends on the internal resistances of battery and voltmeter
    - e.g. the indicator participates in acid-base titrations
    - e.g. dilution and volume displacement effects in potentiometry
- To the extent that such phenomena occur your report line has to specify
- the measurement principle
  - the presence/or absence of a correction

Failing to define the measurand accurately is a source of (definitive) uncertainty.



## 2. Understanding Measurement

**measurand:** particular quantity *intended to be measured*

e.g. HbA1c

surrogate measurement for a surrogate parameter of multiple index-conditions

1. goodness of Glc-control
2. pathophysiological hypothesis: Glc as a driver of Glc-dependent effects
  - Glc- and insulin-desensitization & -irresponsiveness  
(hormonal & metabolic)
  - blood vessel- and nerve-damage
  - ...
3. indicator of prognosis

(Brownlee, *Diabetes* 54:1615-25, 2004)

Failing to educate the user about the use of the results is a source of error.

## 2. Understanding Measurement

### measurement

process to experimentally obtain an estimate of the magnitude of a **quantity**

1. the **measurement procedure** can be described and analyzed as a model
2. measurement presupposes a calibrated **measuring system**, preferably subsequently verified (iQC)

Often we limit ourselves to analysis of this component,  
as we have direct access to it within the confinement of our labs.

This attitude neglects significant pre- and post-analytical steps of the total measurement process.

## 3. Understanding Error

### precision and accuracy of experimental data

#### precision

reproducibility of results or the agreement between the numerical values for repeated measurements made under identical measurement conditions

1. always has a positive sign
2. measured as average deviation from the mean (or median), or as the spread or range, or as the SD on repeated measurements, ...
3. can be expressed absolute or relative as fraction of the mean (or median)

#### trueness - accuracy

nearness to the *accepted* value (as the true value is generally not known)

1. can have a positive or negative sign
2. can be expressed absolute or relative as fraction of the mean (or median)

### 3. Understanding Error

**determinate error vs. indeterminate error**  
**bias or systematic error vs. random error**

**error**

difference between a measurement and the accepted value

**determinate errors**

1. arise from methodological flaws, and **in principle can be accounted for**
  - Instrumental errors: imperfect tools
  - Method errors: slowness of reactions, non-specific reactions, interferences, ...
  - Personal errors: **ignorance, carelessness**, prejudices, physical limitations, ...
2. have a definite value = **bias, systematic error**

**indeterminate errors**

1. arise from **uncontrollable variations**
2. produce scatter about a mean value = **random error, imprecision**

Call it by its name, but also :

Is method robust ?

Can you simplify procedure ?

## 4. Understanding Uncertainty of Measurement

### measurement uncertainty

(cited from info circulating in CLSI C51-P working group, unpublished communication)

parameter, associated with the result of a measurement, that characterizes the **dispersion** of the values that could **reasonably** be attributed to the measurand  
(VIM93, ISO-15189 2003 3.17)

The parameter may be, for example, a standard deviation, (or a given multiple of it) or the half-width of an interval having a stated level of confidence.  
(ISO 15195)

### revised definition

(cited from info circulating in VIM working groups, unpublished communication)

... *based on relevant available information*

This dispersion is due to

**definitional uncertainty** of the measurand  
**random** and **systematic** effects in the measurement

A restrictive definition of UM, will create little or no value.

By contrast, the **revised definition addresses key issues**

## 4. Understanding Uncertainty of Measurement

### **metrological traceability**

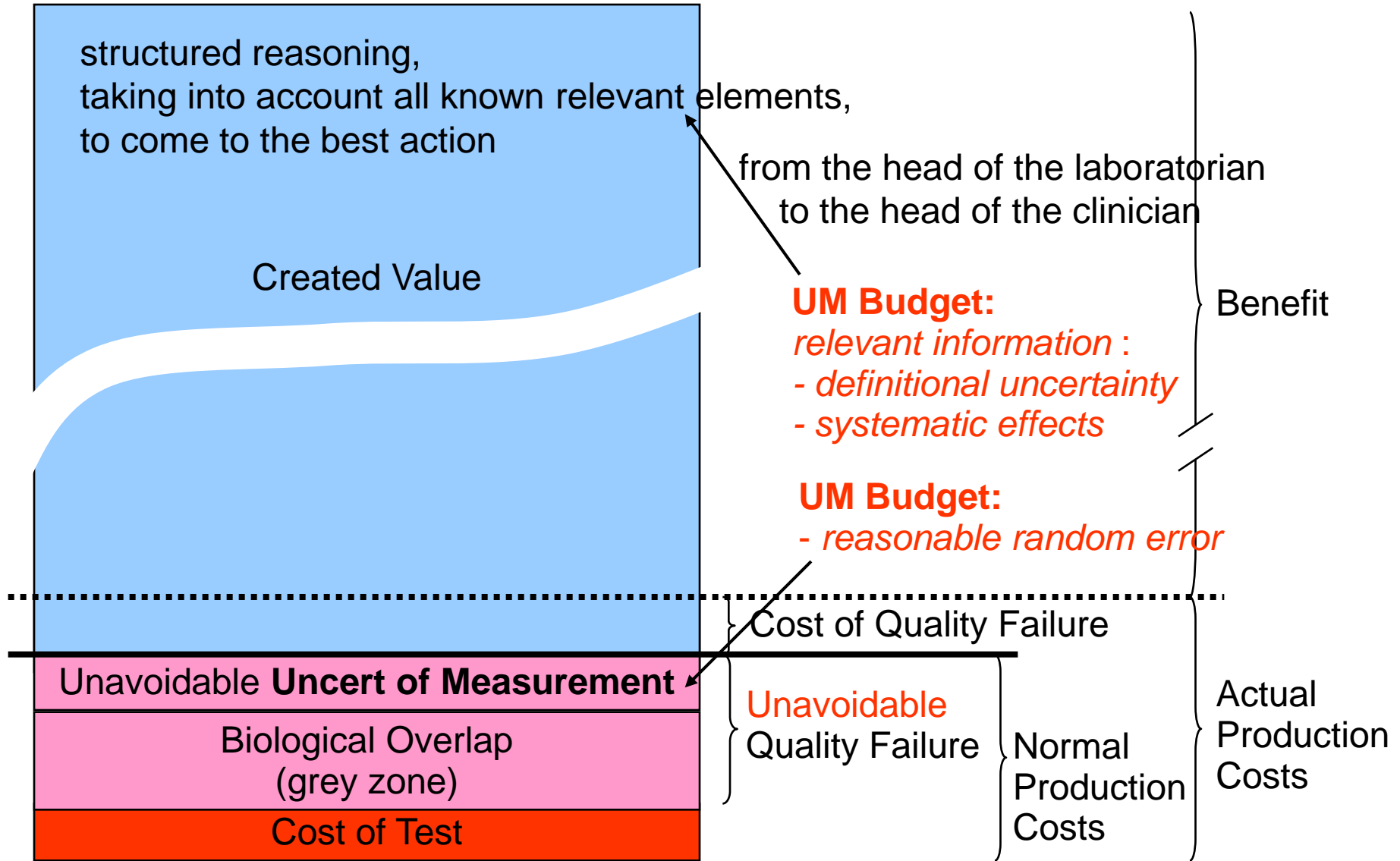
property of a **measurement result** relating the result to a stated metrological reference through an unbroken chain of **calibrations of a measuring system** or comparisons

1. Each step in the chain contributes to the **measurement uncertainty** (extended VIM definition)
2. Results from a harmonized measurement system support the establishment of common reference intervals and decision limits, which allows for global application of clinical study findings.
3. Metrological traceability is facilitated by expressing results in common units, producing a measurement system that supports shared reference intervals and decision limits.

**Here knowledge about UM creates value**

# Does analytical performance affect clinical utilization ?

## Where can knowledge about UM create value ?



## 5. Understanding GUM (Guide to Uncertainty of Measurement)

Part of the Analytical Test **Validation** :

**Systematic** evaluation and appraisal of (analytical test) validity

(Analytical Test) **Validity** :

No measurement is possible without uncertainty

What are the **components** of that uncertainty ?

**Size** of that uncertainty ?

**Importance** of that uncertainty ?

**fit for purpose = function of the intended use**

Analysis:  
**Design for Quality**

Proper starting & end point  
of this **think-experiment**



## 5. Understanding GUM

### Methodology

- Techniques for estimating dispersion and bias  
(in classical GUM, the latter is neglected)
- Specification of the measurement
- Identification of the sources (components) of uncertainty
- Quantification of these sources (in groups)
- Combination of quantified groups

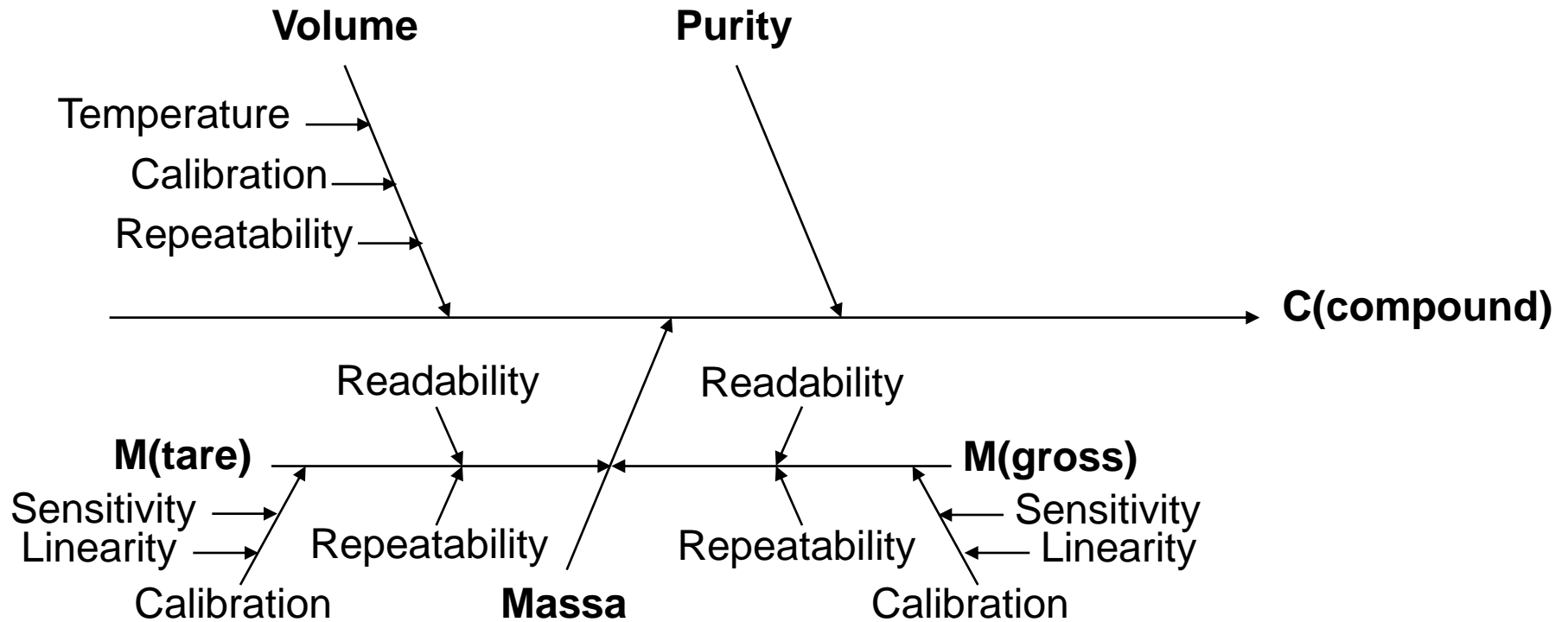
### Sources of information

- data accumulated during validation and verification of a test
  - prior to its application in testing environment  
(implementation validation, use of certified reference materials, ...)
  - **during the life cycle of the method:**
    - accumulated data of internal quality control (iQC)
    - inter laboratory studies, proficiency test schemes

## 5. Understanding GUM

Building models:

e.g. Calibrator formulation



The analysis resembles Fault Trees, treated in the Teaching Module on Risk



## 5. Understanding GUM

**Quantifying uncertainty** (mainly, reasonable dispersion):

### **Type A:**

by the statistical analysis of a series of repeat observations of the measurand obtained under specified measurement conditions.

### **Type B:**

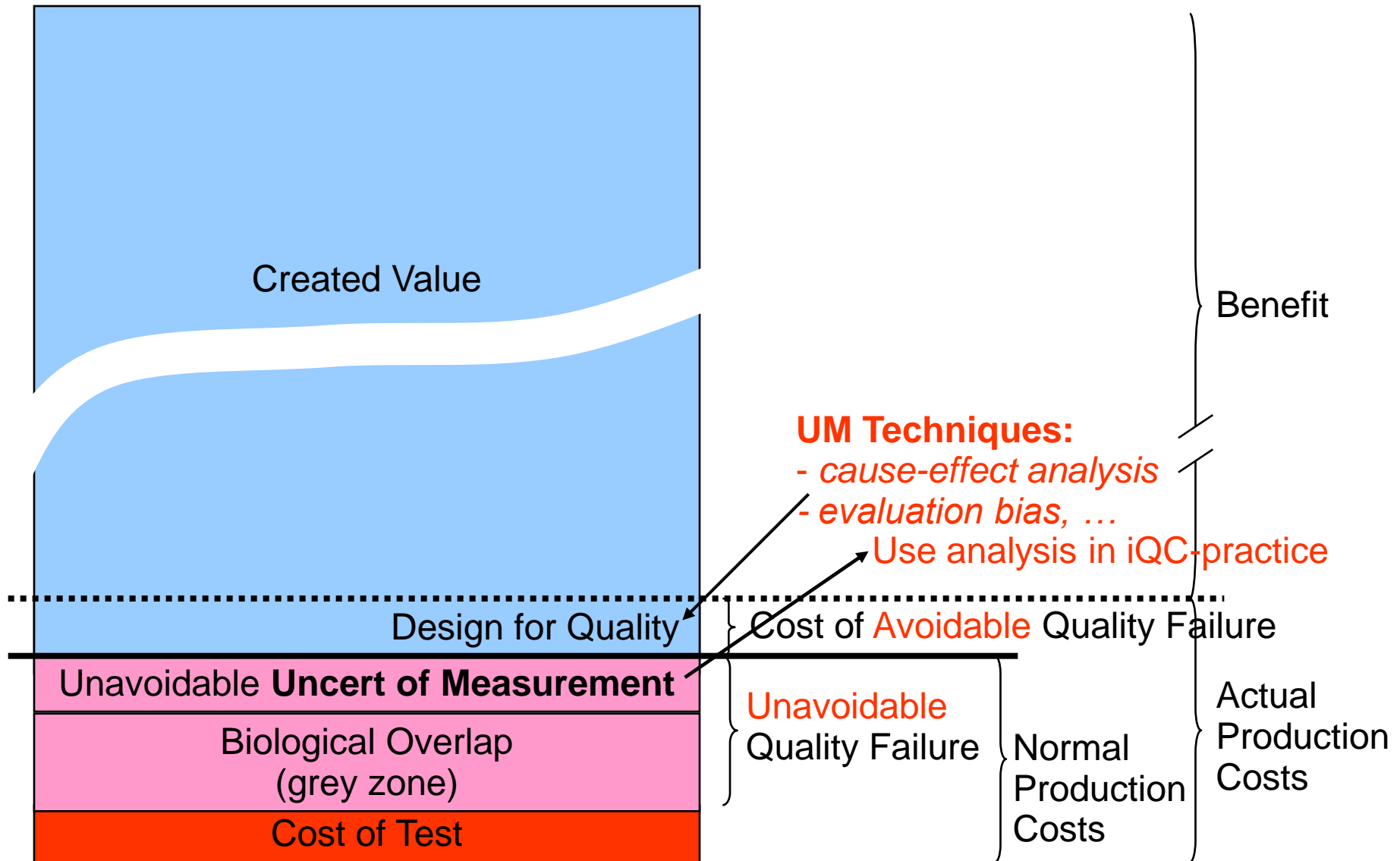
based on available information

- previous measurement data
  - manufacturer's specifications
  - data provided in calibration certificates
  - uncertainties assigned to reference materials
- etc....

- Exploit your iQC-data
- Use UM-analysis to your advantage in iQC-practice

# Does analytical performance affect clinical utilization ?

## Where can knowledge about UM create value ?



# Nuts and Bolts

## General Relationship between the combined standard uncertainty and the uncertainty of the component quantities

### First order Taylor Series approximation of the measurement process

$$u^2(y(x_1, x_2, \dots)) = \sum_{i=1}^N \underbrace{\left(\frac{\partial y}{\partial x_i}\right)}_{\text{sensitivity}} u_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \underbrace{\left(\frac{\partial y}{\partial x_i}\right) \left(\frac{\partial y}{\partial x_j}\right) c_{i,j}}_{\text{Covariance: } u_i u_j r_{ij}}$$

For independent parameters, the second term is absent.

## Estimating SD from available information:

### Gaussian distribution:

reading is within  $\pm n$  mg with 95% confidence

$$SD = n / 1.96 \text{ mg}$$

### Uniform distribution:

Grade A volumetric flask certified within  $\pm n$  ml

Extreme readings judged likely : model corresponds to a uniform distribution

$$SD = n / \sqrt{3} \text{ ml}$$

### Triangular distribution:

Grade A volumetric flask certified within  $\pm n$  ml

Experience learns extreme readings unlikely : model corresponds to a triangular distribution

$$SD = n / \sqrt{6} \text{ ml}$$

## Propagation of determinate error: Sum or Difference

$$y = a + b - c \quad (1)$$

$$(y + \Delta y) = (a + \Delta a) + (b + \Delta b) - (c + \Delta c) \quad (2)$$

$$\Delta y = \Delta a + \Delta b - \Delta c \quad (2-1)$$

The absolute error of a sum is the algebraic sum of the absolute errors

## Propagation of determinate error: Product

$$y = a b \quad (1)$$

$$(y + \Delta y) = (a + \Delta a) (b + \Delta b) \\ ab + a \Delta b + b \Delta a + \Delta a \Delta b \quad (2)$$

$$\Delta y = b \Delta a + a \Delta b + \Delta a \Delta b \quad (2 - 1)$$

$$\Delta y / y = \Delta a / a + \Delta b / b + \Delta a \Delta b / ab \quad (2 - 1) / 1 \\ \approx \Delta a / a + \Delta b / b$$

The relative error of a product is the sum of the relative errors  
and *mutatis mutandis*

$$y = a / b$$

$$\Delta y / y \approx \Delta a / a - \Delta b / b$$



## Propagation of error: Exponential Calculations

$$y = a^n \quad (1)$$

with n (power or root), containing no uncertainty

$$dy = n a^{n-1} da \quad (2)$$

$$\begin{aligned} dy / y &= n (a^{n-1} / a^n) da && (2 / 1) \\ &= n (da / a) \end{aligned}$$

The relative error of the power of a is exponent-times the relative error of a.

## Propagation of error: Logaritms

$$y = \text{Log } a = 0.434 \ln a \quad (1)$$

$$dy = 0.434 da / a \quad (2)$$

## Propagation of error: Antilogaritms

$$a = \text{antiLog } y \quad (1)$$

$$dy/0.434 = da / a \quad (2)$$

The absolute error of the logarithm of a is proportional to the relative error in a, and conversely for the antilogarithm.

## Accumulation of indeterminate error

### **Addition and subtraction:**

The errors accumulate with a random algebraic sign.

A pythagorean rule applies to the absolute standard deviations

### **Multiplication and division:**

The errors accumulate with a random algebraic sign.

A pythagorean rule applies to the relative standard deviations (CV's)

### **Exponential calculus:**

Here the sign of the error is necessarily identical throughout the multiplications, and thus the pythagorean rule does not apply.

The CV of a power is exponent-times the CV of the component quantity.

# Significant Figure Convention

## Sum

$$\begin{array}{r} 123.45 \\ + 1.2345 \\ \hline 124.68 \end{array}$$

The last significant position of the result is determined by the last decimal position in the component quantity with the worst precision.

## Product

$$\begin{array}{r} 3.33 \\ \hline \times 3.0001 \\ 10.0 \end{array}$$

The number of significant positions in the result is determined by the component quantity with the least number of significant figures.

## Logarithm

$$\log 3.00 * 10^{-3} = -2.523$$

The characteristic (to the left of the digital point) locates the digital point. The mantissa (to the right of the digital point) has the same number of significant numbers as the original quantity.

## Aleatory and epistemic uncertainties

**Aleatory uncertainty** : uncertainty induced by randomness

**Epistemic uncertainty** : uncertainty induced by imperfect knowledge about something that in principle is knowable

### Basic example :

$$y = y_0 + S_W p + S_B q \quad (\text{e.g. true mean } \pm \text{ within- and between-batch variance})$$

$y_0$ ,  $S_W$ ,  $S_B$  Epistemic variables  
(imperfectly known expert estimates  
for the true mean, and between and within variability)

$p$ ,  $q$  Aleatoric variables  
(zero-mean unit-variance random variables)

Test-drive simulator: accumulation of internal quality control data 

### Extended applications :

Any calculated parameter making use of estimated constants

When the confidence interval of such estimates is known, it can enter the model.

# Internal Quality Control (iQC)

Statistical Process Control is treated in detail in a separate teaching module



# Use of iQC data

## Eliminate a misconception about bias

In iQC follow up you see layers of data points, jumping to new levels after a calibration event, new reagent lot, etc.

This is **NOT BIAS BUT RANDOM ERROR**

- you can not predict the sign of the change

## ANOVA analysis of your iQC data : **simplified UM model**

This corresponds to a Taylor expansion

- indeterminate error evaluated directly  
(with abstraction of sensitivity)
- covariance = 0

Test-drive our simulator 

# Use of iQC data

## Take-home observations from running the simulator

The iQC long-term SD can approximate the combined UM, provided :

- a sufficient number of significant events are accumulated

An accurate estimate of the iQC target (central tendency & dispersion)

- depends on regular updates
- results in these benefits:
  - the false detection rate of the iQC-rule will decline
  - the true detection rate of the iQC-rule depends mainly on the power of the rule (but may also benefit)

ANOVA: identify principal components of variance &

- can method be made more robust ?
- can you simplify procedure ?

# Use of iQC data

## ISO 15189 § 5.6.2.

The laboratory shall determine the uncertainty of results, where relevant and possible. Uncertainty components, which are of importance, shall be taken into account.

Sources that contribute to uncertainty may include sampling, sample preparation, sample portion selection, calibrators, reference materials, input quantities, equipment used, environmental conditions, condition of the sample and changes of operator.

The lab shall have a procedure

### Lab Procedure:

1. For quantitative tests, the UM is available on demand.

At relevant levels, the combined UM is approximated by the long-term iQC imprecision times a coverage factor (1.95).

2. The consultative function of the lab addresses other elements of UM (definitional uncertainty, systematic and spurious errors).



# New Developments

# Probabilistic Model vs Risk Model

## Classical MU : Probabilistic Model

- construct model for all steps in the analytical process
- estimate / assign indeterminate error for each step
- compound the error-propagation
- limited to quantitative results
- reported as a confidence interval

estimate of total dispersion · coverage factor

## Extended MU : Risk Models

- inventory of significant quality failures
  - insufficient analytical quality
  - insufficient specificity
    - analytical non-specificity
    - biological / physiological interfering responses
- estimate extent and frequency of such (spurious) events
- applicable to quantitative and non-quantitative results
- reported as an inventory of pitfalls (= body of knowledge)

# Reasonable Dispersion vs Total Error

Classical MU :

Data model

$$X = X_{\text{accepted}} + e_i * \text{random}(0,1)$$

Reasonable  
variability

Total Error :

Data model

$$X = [X_{\text{true}} + \Delta_j] + e_i * \text{random}(0,1)$$

Bias

Epistemic variables

Fallacy of the total error (bias) model :

While bias does occur,  
as soon as it is known,

it no longer exists (except for its epistemic uncertainty),  
and as long as it is not known,

it can only enter the calculus as a hypothetical.

## Literature

NIST reference on Constants, Units and Uncertainty

NIST-TN 1297 (1994)

The expression of uncertainty in quantitative measurement

EAL-G23 (1996)

Quantifying Uncertainty in Analytical Measurement

EURACHEM / CITA Guide CG4 (QUAM 2000.1)

Introducing the Concept of Uncertainty of Measurement in Testing in Association with the Application of the Standard ISO/IEC 17025

ILAC G17 (2002)

Guide de Métrologie à l'usage des laboratoires d'analyses de biologie médicale (Collège Français de Métrologie)

Uncertainty of Measurement in Quantitative Medical Testing

(Australasian Association of Clinical Biochemists, 2004)

Internet resources

★ <http://physics.nist.gov/cuu/index.html>

★ <http://www.measurementuncertainty.org/mu/guide/index.html>